

**Method for testing the Error Ratio BER of a device
according to Confidence level, Test time and Selectivity**

The invention concerns a method for testing the error
5 ratio BER of a device under test against a specified
allowable error ratio.

The statistical test can be an example of the test of a
digital receiver for example for cellular mobile phones as
10 a Device Under Test (DUT). The test shall decide if the
DUT fulfills a specified Bit Error Ratio (BER) or not. The
true BER is not target of the test.

It is the object of the invention to design an optimal
15 test. The invention derives and exploits the knowledge,
that the test is governed by the 3 parameters above
Confidence level, Test time and Selectivity.

In the state of the art the test could actually be
20 optimised according to only the two parameters Test time
and Confidence level only without awareness of
Selectivity.

In WO 02/089390 A1 the test exploits well known properties
25 of the distributions. If errors occur independently, this
can be mathematically described by the binomial
distribution. If errors occur rarely ($\lim \text{BER} \rightarrow 0$) it can
be described by the Poisson Distribution. The Poisson
Distribution is used to derive pass and fail criteria.

30 With a finite number of samples ns, the final bit error
ratio BER cannot be determined exactly. Applying a finite
ns, a number of errors ne is measured. $ne/ns = ber$ is the
preliminary bit error ratio.

35 In a single test a predefined number of samples ns is
applied and a number of errors ne is measured. ne is
connected with a certain differential probability in the
Poisson distribution. The probability and the position is

not known in the distribution conducting just one single test. Repeating this test infinite times, applying repeatedly the same ns, we get the complete Poisson distribution. The average number of errors is NE. NE/ns is
5 the final BER.

The Poisson Distribution can be defined as

$$dpois(ne, NE) = (NE^{ne}/ne!) e^{-NE} \quad (1)$$

10 The Poisson distribution, shown in Fig. 1, has the variable ne and is characterized by the parameter NE. Real probabilities to find ne between two limits are calculated by integrating between such limits.

15 It has to be noted, that the Poisson distribution is an approximation: Independent error occurrence is described by the binomial distribution. If the BER approaches 0 the Poisson distribution approximates the binomial distribution. Hereafter it is shown, that Poisson
20 Distributions and the Chi Squared Distribution are equal. Just the form is different. On the other hand there are two inverse cumulative operations. One of them, the chi square one, is useful for our purpose.

25 The experiment, the Poisson distribution is based on, is: having observed a certain number of samples ns, the number of events ne is counted to calculate the ratio ne/ns. The experiment, the Chi Square distribution is based on, is: having observed a certain number of events ne, the number
30 of samples ns is counted to calculate the ratio ne/ns. Poisson and Chi Square are valid only if ne<<ns.

The experiment of the Chi Square distribution is always terminated by an event. In contrast the experiment of the
35 Poisson distribution almost never is terminated by an event because of $ne/ns \rightarrow 0$. This explains that the Poisson distribution needs one event more to equal in its form the Chi Square distribution. The equivalence is as follows:

$$\begin{aligned} 2 * \text{dchisq}(2 * \text{NE}, 2 * \text{ne}) &= \text{dpois}(\text{ne}-1, \text{NE}) \\ 2 * \text{dchisq}(2 * \text{NE}, 2 * (\text{ne}+1)) &= \text{dpois}(\text{ne}, \text{NE}) \end{aligned} \quad (2)$$

We have seen: Chi Square and Poisson both describe the
 5 same distribution. Both are functions of 2 variables ne
 (integer) and NE (real). However both are not commutative
 in NE and ne. Hence there are two inverse operations (3)
 and (4):

$$10 \quad D = \int_0^{ne} \text{dpois}(ni, NE) dni = 2 * \int_0^{ne} \text{dchisq}[2NE, 2 \cdot (ni + 1)] dni \quad (3)$$

with: D is the wrong decision probability or confidence level (input)

ni is the integration variable

15 ne is the measured value (input, discrete). It is the integration limit.

NE (real) is tuned such that the integral is consistent.

It returns an NE as a function of the two parameters D and
 20 ne: qchisq (D, ne), qchisq is the inverse cumulative chi square function.

$$D = \int_0^{NE} \text{dpois}(ne, NI) dNI = 2 * \int_0^{NE} \text{dchisq}[2NI, 2 \cdot (ne + 1)] dNI \quad (4)$$

25 with: NI is the integration variable.

NE (real) is the integration limit.

ne (discrete) is tuned such that the integral is consistent.

30 It returns ne as a function of the two parameters D and NE: qpois (D, NE), qpois is the inverse cumulative Poisson function.

Our target requires equation (3). This is usually called
 35 the Inverse Cumulative Chi Square function. Equation (4) is the solution for another target. This is usually called the Inverse Cumulative Poisson function. (3) returns a

greater NE than (4) returns with respect to ne. (3) returns a continuous NE, (4) returns a discrete ne.

An introduction into the notion of confidence range is
5 explained as follows. In a single test ns samples and measure ne errors are applied. The result can be a member of different distributions each characterized by another parameter NE. We ask for two of them:

10 1) The worst possible distribution NE_{high} , containing our measured ne with [D= e.g. 0.0085%] probability in the sense:

$$0.000085 = \int_0^{ne} dpois(ni, NE_{high}) dni \quad (5)$$

with: ni is the integration variable

15 ne is the measured value

NE is the variable to tune in order to make the integral consistent.

The result of the inverse cumulative operation is NE_{high} .

20 2) The best possible distributions NE_{low} , containing our measured ne with [D=0.0085%] probability in the sense:

$$0.000085 = \int_{ne}^{\infty} dpois(ni, NE_{low}) dni \quad (6)$$

The result of the inverse cumulative operation is NE_{low} .

25 To illustrate the meaning of the range between NE_{low} and NE_{high} the following may be helpful: In the case the measured value is ne, the final mean value NE can be found with high probability (100% - 2*0.0085%) in the range from 30 NE_{low} to NE_{high} , called confidence range.

The inverse cumulative Chi Squared distribution gives the wanted results:

Inputs: number of errors ne, measured in this test.

35 Probability D and the complementary probability
1- D

Output: NE, the parameter describing the average of the distribution.

An example can be found in Fig. 2.

- 5 Same as the width of the distributions the confidence range increases proportional to SQR ne, which means, it increases absolutely, but decreases relatively to the measured number of errors.
- 10 In this section the notion of the confidence range is used to calculate the early pass and early fail limit.

If the entire confidence range is found, calculated from a single result ne, on the good side of the specified limit
15 it can be stated: With high probability 1-D, the final NE is better than the limit.

If the entire confidence range is found, calculated from a single result ne, on the bad side of the specified limit
20 it can be stated: With high probability 1-D, the final NE is worse than the limit.

Transcription of the above text into formulas lead to the following. The current number of samples ns is calculated
25 from the preliminary ber and the preliminary ne with:

$$\text{ber} = \text{ne}/\text{ns} \quad (7)$$

$$\text{BERlimit} = \text{NE}_{\text{limit}} / \text{ns} \quad (8)$$

for abbreviation in the formula: $\text{ber}_{\text{norm}} = \text{ber}/\text{BER}_{\text{limit}} = \text{ne}/\text{NE}_{\text{limit}}$ (normalized ber)

- 30 Early pass stipulates: $\text{NE}_{\text{high}} < \text{NE}_{\text{limit}}$
- Early fail stipulates: $\text{NE}_{\text{low}} > \text{NE}_{\text{limit}}$

An example of the early fail and the early pass limit are displayed in Fig. 3. According to Fig. 3 an unfavourable
35 test can last infinite time, when the measured BER stays in between the limits. This requires an artificial stop condition. Whichever ne is proposed as a final stop condition, e.g. ne=200, the test can leave the area between the Early Pass and the Early Fail limit through

the open end of the right side of Fig. 3. This situation needs an arbitrary pass or fail decision, e.g. pass, if the test hits the vertical 200 error line. This situation has the following drawback: The test has two different qualities. A good one with wrong decision risk D, when the test hits an early pass or early fail limit, and a worse and variable one, when the test hits the vertical 200 error line. This is variable and is depending on the height, it crosses the line. The quality of the test in terms of wrong decision probability is variable in the range D to 50%. As already proposed in WO 02/089390 A1 the situation can be improved by a better trade-off: Instead a test with different qualities against one limit, we design a test with a fixed uniform quality against two limits.

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WO 02/089390 A1 preserves the definition of the early fail limit: A DUT is failed and the probability of $D = 0.0085\%$ is accepted, that is actually better than the limit.

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WO 02/089390 A1 proposes a meaningful redefinition of the early pass limit: A DUT is passed and the probability of $D=0.0085\%$ is accepted, that is actually worse than M times the limit ($M>1$). M is the Bad DUT Factor.

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An example for $M = 1.5$ is shown in Fig. 4. This produces the following consequences:

- The early pass limit is shifted upwards by the factor of M.

- The early fail and the early pass limit intersect.

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- The intersection coordinates are the normalized test limit and the maximum number of events.

This leads to the following

$\text{berlim}_{\text{bad}}^{\text{pass}}$: early pass limit against the bad DUT limit

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$\text{berlim}_{\text{fail}}$: early fail limit against the specified limit.

In WO 02/089390 A1 a pass and a fail limit for any instant in the test is calculated. However the decision is only

allowed at one predefined instant in the test. The wrong decision risk for a single test step is D. However it is desirable to have a predefined wrong decision risk F for the entire test. The approach to derive F from D is
5 explained here and is discussed in PCT-application PCT/EP03/00970 not published before the priority date of this application.

Provided a single BER trajectory with final BER on the
10 limit hits the early fail limit. If a fail is decided at this instant of the test, the wrong decision risk is as small as D. For each member of a large population of DUTs a wrong decision can happen, with probability D, accumulating to an amount $F > D$ for the entire population.
15 D is the wrong decision probability based on the statistical totality of samples with BER on the limit. F is the wrong decision probability based on the statistical totality of DUTs with BER on the limit. The same holds for a bad DUT, hitting the early pass limit.

20 We call D the wrong decision probability at a single test step and F the wrong decision probability for the entire test. For a real test it is desirable to define in advance the wrong decision probability F of the entire test. An
25 exact theory is not available for this problem. It is proposed to derive D from F by the following simulation: A large population of DUTs with BER on the limit is simulated and decided against the early pass and early fail bound, with a free D-parameter in the early pass and fail limit. The simulation will show, that a certain fraction F ($D < F < 1$) falsely fails. The complementary simulation is: A large population of DUTs with M^*BER (bad DUT) is simulated and decided against the early pass and early fail bound, with a free D-parameter in the early pass and fail limit. The simulation will show, that a certain fraction F ($D < F < 1$) falsely passes. Both false decision fractions are approximately equal and represent the wrong decision probability F for the entire test. D is
30
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tuned such that F corresponds to the predefined wrong decision probability.

In the present state of the art the distribution of errors
5 is considered in the totality of samples for one DUT. This assumes that the error events occurred independently and the BER is time independent. Furthermore, a distribution function is applied which is valid only for small BER ($\lim \rightarrow 0$). From the statistics of errors in the totality of
10 samples the theory returned a pass and a fail limit dependent on the wrong decision probability D ($0 < D << 1$). The pass and fail limit can be calculated at any instant of the test (ordinal of error), however it may be used for decision just once at a predefined instant. The result at
15 this instant can be pass, fail or undecided. The meaning of D is: One DUT is measured by determining BER at the predefined instant. If the BER at the predefined instant hits the pass \ fail limit, it is decided accordingly and a wrong decision probability D is accepted.

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If BER is beyond the pass \ fail limit, the wrong decision probability is $< D$ and we decide accordingly too. If BER is inside the pass \ fail limits the wrong decision probability is $> D$ and we cannot decide with the wanted
25 quality. This method is not usable for test time reduction, however it is the basis for this.

In the not prepublished application PCT/EP03/00970 a method is proposed to apply an early pass and an early
30 fail limit at any instant of the test. It demands the following definition for the wrong decision probability:

A DUT is measured by determining a series of preliminary BER (t): $BER(t) = ne(t)/ns(t)$. t=0 is the start of the
35 test. If the preliminary BER at any instant of the test hits the early pass \ early fail limit, it is decided accordingly and a wrong decision probability F ($0 < F << 1$) is accepted. The method to achieve the target was the following: A large number of DUTs with equal BER has been

simulated, passed and failed against the pass fail limits derived with the above discussed. D has been tuned down, until a fraction F of the DUTs behaved according to the new definition ($0 < D < F < 1$).

5

The notion of BAD DUT Factor $M > 1$ was introduced as follows: A bad DUT is normally failed, however it may falsely pass with a wrong decision probability F. A good DUT on the specified limit is normally passed, however it 10 may falsely fail with a wrong decision probability F. With the bad DUT factor M it could have been ensured that the early fail limit and the early pass limit intersect and the test was reliably finalised after a finite time.

15 However there are still the remaining drawbacks in the present state of the art:

- There occurs a deformation of the distribution. If a DUT is decided pass or fail, it leaves the population of 20 all DUTs and the remaining distribution is deformed. This deformation was not considered.

- The shape of the pass and fail limit at each point of the test was calculated from the theory, applicable at 25 just one point of the test. This shape was preserved when making the pass fail limits applicable at each point of the test. This gives away the possibility to shape the pass and fail limits for further optimisation.

30 - Early pass and early fail limit are not independent. The early pass and the early fail limit are calculated independently without proving that this is justified. However in the measurement process they are used simultaneously. In reality the early pass and early fail 35 limit are not independent. So the such calculated early pass and fail limit do not completely represent the reality of measurement.

- As stated above the notion of Bad DUT factor is a trade-off between drawbacks. In addition the bad DUT factor M is not acceptable in any case. Specified Bit Error Ratios are in the order of magnitude 0.1 % to 10 %.
5 Assuming a bad DUT Factor M=1.5 the equivalent bad DUTs are 0.15% to 15%. This is acceptable. Data throughput in a communication system is also of statistical nature and must be measured and decided pass or fail. The specified lower limit of throughput related to the ideal throughput
10 in different throughput tests is in the range of a few % to near 100%. To make it comparable with BER, we define the complement of the relative throughput: lost relative throughput. Complementary this is in the range of near 100% down to a few %. For e.g. lost relative throughput =
15 1%, the currently used M=1.5 is highly meaningful. For e.g. lost relative throughput = 99%, the currently used M=1.5 is obviously meaningless.

- In the state of the art the early pass and early fail
20 limit is based on just one, the following definition: A bad DUT may falsely pass with a wrong decision probability F and a good DUT may falsely fail with a wrong decision probability F. There are other, highly effective, definitions possible as well.
25

It is the object of the present invention to provide an test time optimized method for the statistical test for a device under test with a high selectivity, i. e. the device under test with a specific error ratio shall be
30 passed with a specific probability, while a slightly worse device under test shall be failed with a high probability.

The object is solved by the features of claim 1.

35 According to the solution of the present invention the early pass limit is constructed by using an empirically or analytically derived distribution for a specific number of devices each having the specified allowable error ratio by separating a specific portion of the best devices from the

distribution for a specific number of erroneous samples and proceeding further with the remaining part of the distribution for an incremented number of erroneous samples.

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The dependent claims concern further developments of the invention.

10 The invention is further described with respect to the drawings. In the drawings

Fig. 1 shows an example for a Poisson distribution curve;

15 Fig. 2 shows an example for a confidence level;

Fig. 3 shows the early pass and early fail limits;

20 Fig. 4 shows an early pass and early fail limit with bad device factor M;

Fig. 5 shows an example of the measurement practice;

25 Fig. 6 shows an example of a simulation of devices under test;

Fig. 7 shows an example for an early pass and early fail limit;

30 Fig. 8 shows a visualisation of selectivity;

Fig. 9 shows another visualisation of selectivity and

Fig. 10 shows modifications of pass and fail curves.

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A statistical test, for example the test of a digital receiver as a Device Under Test DUT, is discussed hereafter. The test shall decide if the DUT fulfills a

specified Bit Error Ratio BER or not. The true BER is not target of the test.

The test has 3 dimensions of quality:

- 5 - The decision shall be done correctly with high probability. This corresponds to a low wrong decision risk.
- The test shall be decided pass or fail after as few samples as possible. This corresponds to short test time.
- The selectivity shall be high, i.e. a DUT with the specified BER shall be passed with the above mentioned probability, while a slightly worse DUT shall be failed with high probability. Note, that 15 test quality and DUT quality have to be distinguished.

The preliminary BER is continuously determined and entered into the diagram as shown in Fig. 5. When the BER 20 trajectory intersects the pass or fail limit, the DUT is decided accordingly.

For the invention literally the classical concept of probability is used: When P% of an infinite large 25 population of DUTs is hit from a statistical event, then an individual of this population is hit by this event with a probability of P%. Hence it is discussed, simulated and calculated at a very large population of DUTs, to conclude for the probability that an individual is hit by 30 the statistical event. If 99% of the population of DUTs, all with BER on the specified limit, hit the pass limit during the test, and 1% hit the fail limit, then 1 DUT with BER on the limit is decided pass with 99% probability and fail with 1% probability.

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To construct the pass fail limits, infinite many DUTs are simulated as shown in Fig. 6. The numbers in the table are created using a pseudo random generator repeatedly according to the following:

Random number [0...1]
Above true BER: correct
Below true BER: error
Count until the first (next) error occurs
5 Enter the count into the array of Fig. 6

The interpretation is as follows: The pseudo random generator represents the BER characteristics of the DUT. Errors occur independently and the true BER is time 10 independent.

One line represents the preliminary BER(ne) of a single DUT. BER(ne) is the ordinal of error at this instant divided by the sum of samples from the left to this 15 instant, with ne = number of errors. This can be better understood from the following example.

Example: BER at the first error: 1/666
BER at the second error: 2 / (666+902)
20 BER at the third error: 3 / (666 +902+ 1555)

The set of all rows represent the BER(ne) evolution of the population of DUTs. The left column represents the BER (ne=1) of all DUTs when the 1st error occurs. This can be 25 obtained from the inverse number of samples. The neth column represents the BER (ne) of all DUTs when the neth error occurs.

Other BER characteristics can be simulated with 30 corresponding generators: e.g. we can re-use a modified form system simulations, deriving the receiver BER (including channel decoder) under fading conditions, creating non independent and time variant bit errors.

35 The first point of the early pass limit is constructed by the following rules: Take the left column and separate the DD (e.g.1%) best from the (1-DD) (e.g.99%) worst DUTs. The BER value at the separation point of the worst of DD best DUTs marks a point of the early pass limit. The best DUTs

are from now on regarded as pass and leave the population and the worst DUTs are regarded as undecided and are processed in the same way to construct the next point of the early pass limit by using column 1 with column 2.

5

As an example for the first point of the pass limit separating the 25% best DUTs form the 75% worst ones gives the first point of the early pass limit: BER=1/9007

10 The second point of the early pass limit is constructed as follows: Using column 1(left) with column 2, however disregarding all lines with already passed DUTs we repeat the process. We separate the DD(e.g.1%) best from the (1-DD) (e.g.99%) worst DUTs. The BER value at the separation
15 point marks the second point of the early pass limit. The best DUTs are from now on regarded as pass and leave the population and the worst DUTs are regarded as undecided and are processed in the same way to construct the next point of the early pass limit.

20

This way an early pass limit EPL in Fig. 7 is constructed (controlled) and an early fail limit EFL in Fig. 7 gets implicitly (uncontrolled). The horizontal line represents the true BER. This is the basis for construction. The
25 early pass limit may intersect the true BER due to deformation of the distribution. The good decided DUTs leave the distribution and shift the undecided part distribution towards the bad direction.

30 In the construction, a certain DD is selected (e.g. 1% of the current undecided population) and the separation process until FF is repeated until (e.g. 99% of the initial population) is decided pass. The undecided rest is declared fail. This guarantees a limited test time. In
35 this example this needs 229 steps.

FF is the pass probability for the entire test. DD is the pass probability for a single test step. DD is selected so that it passes a constant partition of the remaining

undecided population. There are many possibilities for DD for one target FF. A smaller DD is used and the separation process is repeated more often to reach the final pass probability, e.g. separate 0.5% per step. This needs 459
5 steps to separate 99% of the initial population. Obviously the test keeps a pass probability at 99%. Some test time is lost. The quality, gained by this trade-off, is selectivity.

10 A DUT on the specified BER limit passes with 99% probability according to the construction. Using the constructed pass and fail limits and applying them to a DUT, which is a little Δ higher in BER than the specified limit, the pass probability for that DUT can be
15 determined, which is a little Δ lower than the specified limit. The ratio (Δ pass probability) to (Δ BER) is the selectivity of the test. A visualisation of selectivity is shown in Fig. 8 and Fig. 9.

20 In general two of the three parameters number of samples, correct decision probability and selectivity can be predefined. The third one can be derived.

25 The number of samples NS is minimized in the following sense: The maximum number of samples in the test is limited and the actual number of samples is dependent on the quality of the DUT.

Following relationship applies:

30

$$\text{Mean NS (better)} < \text{Mean NS (specified limit)} < \text{Max NS} \quad (9)$$

35 Max NS is the maximum number of samples. This number can be predefined or derived.

Mean NS (specified limit) is the mean number of samples in a test, when the DUT is on the specified limit. The actual number of samples in an individual test is not

predictable. Mean NS (better) is the mean number of samples in a test, when the DUT is better than the specified limit. The actual number of samples in an individual test is not predictable, however the smaller, 5 the more extreme the quality of the DUT. This is a strong motivation to design DUTs better than the specification, as it saves test time.

A DUT on the limit shall be passed with FF% probability. 10 FF near 1 and correct with respect to pass is the conventional understanding of correct decision probability. However the principal can be used for any FF (0<FF<1) and it is free to understand correct with respect to fail. To optimize test time, we should construct 15 (control) the pass limit and get the fail limit implicitly (uncontrolled), and not vice versa. The reason is the following: By nature a bad DUT is decided faster than a good one, see Fig.7 at constant test time. So it is advantageous to optimize for a good DUT and disregard any 20 optimisation for the bad DUT which is decided faster anyhow. If it is free to trade FF against the specified BER-limit, it is optimal for test time, to select an FF near 1. The reason is that this way the majority of the test is controlled and optimized and the minority is left 25 uncontrolled.

The concept of selectivity describes the behaviour of the test, if the DUT deviates from the specified limit. As defined above, selectivity is a differential. However it 30 can be used as a ratio of differences.

Selectivity is approximately

$$\text{Selectivity} = \frac{\text{pass probability} - (\text{the complement of the pass probability, which is the fail probability})}{(\text{Bad DUT BER} - \text{Specified DUT BER})}. \quad (10)$$

35 This concept corresponds approximately to the former concept of Bad DUT factor M.

DD is the pass probability for a single test step. DD is the parameter which controls the test in all parameters:

Correct Decision Probability, Test Time and Selectivity. Constant DD means relative equal separation of good DUTs from the remaining undecided population and means approximately:

5

$$\text{Number of good decided DUTs (ne)} = A/s * e^{-ne/s} \quad (11)$$

with:

10 A: A parameter proportional to the initial count of the population.

ne: number of errors, the variable in this function

s: a parameter proportional to selectivity

This function often occurs in natural processes. It is
15 felt to be optimal for the following reasons: More decisions at the beginning of the test at the expense of less decisions at the end of the test is good for test time but not fair. The notion of uniform selectivity gets
meaningless. Less decisions at the beginning of the test
20 at the expense of more decisions at the end of the test is bad for test time and is of no use, except if there are external constraints, e.g. no decision is allowed unless a minimum test time is elapsed. Then the concept is applicable after minimum test time is elapsed.

25

The pass fail limit PFL, shown in Fig. 10, was discussed above. The separation process was done only from the good side of the distribution. It is optimal fast for the 99% pass decided DUTs and is superior over the other curves
30 for better DUTs. No optimisation effort is done for those 1% DUTs, which are decided fail. It is inferior for worse DUTs.

A compromise of test time with respect to pass and fail
35 is done for the compromise curve CC in Fig. 10. The separation process is done from the good side and the bad side of the distribution simultaneously. E.g. 99% hit the pass curve, e.g. 0.5 % hit the vertical fail line, 0.5% hit the fail curve. The 99% pass decided DUTs are decided

a little bit slower. However the fail decided DUTs are decided a little bit faster in average.

This optimisation of test time with respect to pass and
5 fail is driven into extreme with the extreme curves EC in
Fig. 10. The separation process is done from the good side
and the bad side of the distribution simultaneously. E.g.
99% hit the pass curve, 1% hit the fail curve. The 99%
pass decided DUTs are decided a little bit slower compared
10 to curve CC. However the fail decided DUTs are decided a
little bit faster in average compared to curve CC. These
curves have the disadvantage that the maximum number of
samples is not limited although the average number of
samples is optimized. Hence there is an infinite small
15 probability that an unfavourable test lasts infinite time.

All three variants show that the pass and fail limits are
not independent. The optimal curve OC in Fig. 10 seems
optimal for both pass and fail. The separation process is
20 done from the good side, constructing the pass limit, and
then, in a second run, from the bad side constructing the
fail limit, whereas the implicitly given fail limit (1st
run) and the implicitly given pass limit (2nd run) are not
used. This can be a practical solution. Although the pass
25 and fail limits are not really independent, they are
constructed as if they were independent. Therefore this
solution is theoretically not fully justified and on the
risky side. All curves have the free parameter
selectivity.

30

The above mentioned description implies simulation to get
the pass fail limits. This needs infinite computational
effort for exact results. In case the BER characteristic
of the DUT is given by independent error occurrence and
35 time independent true BER, we give an analytical method.
The computational effort is still high but not infinite
for exact solution.

The left column of the array shown in Fig. 6 represents the BERs ($ne=1$) of all DUTs when the 1st error occurs, with BER = inverse number of samples. The second column of the array shown in Fig. 6 represents the BERs ($ne=2$) of all 5 DUTs when the 2nd error occurs. $BER(ne=2) = 2$ is divided by the horizontal sum of samples from the left to including column 2. The ne^{th} column represents the BERs (ne) of all DUTs when the ne^{th} error occurs. $BER(ne) = ne$ is divided by the sum of samples from the left to 10 including column ne.

This facts can be described analytically: The numbers in the left column follow the geometrical distribution:

$$15 \quad P_1(ns) = BER \cdot (1-BER)^{ns-1} \quad (12)$$

with

BER: true BER of the DUT

20 P_1 : probability to find the first error after ns samples, the variable in the distribution

This distribution is already suited to construct the first point of the early pass limit: Separate the DD best from the (1-DD) worst part of the distribution. The BER value 25 at the separation point marks the first point of the early pass limit. The part-distribution of the best DUTs is from now on regarded as pass and the part-distribution of the worst DUTs is regarded as undecided. It is called $U_1(ns)$ and is furtheron processed. U stands for undecided.

30 The horizontal sum of column 1 and 2 in Fig. 6 is represented by the following distribution

$$35 \quad P_2(ns) = P_1(ns) * P_1(ns) \quad (13)$$

with

P_2 : probability to find the second error after ns samples

*: convolution

$P_2(ns)$ is not directly usable to construct the 2nd point of the early fail limit, but it is

$$T_2(\text{ns}) = U_1(\text{ns}) * P_1(\text{ns}) \quad (14)$$

with

$T_2(ns)$: probability to find the second error after ns samples regarding the loss of the best DUTs from the previous step. T stands for transient.

The $T_2(ns)$ part-distribution is suited to construct the second point of the early pass limit as follows: Separate the DD best from the (1-DD) worst part of the distribution. The BER value at the separation point marks the second point of the early pass limit. The part-distribution of the best DUTs are from now on regarded as pass and the part-distribution of the worst DUTs are regarded as undecided. It is called $U_2(ns)$ and is the basis for the next step. This way we can construct the early pass limit point by point.

Even though the invention is described above with reference to the examples according to the accompanying drawings, it is clear that the invention is not restricted thereto, but it can be varied in several ways within the scope of the inventive idea.